The Role of Opportunity to Learn and School Socioeconomic Composition in Reducing Racial and Gendered Disparities in Mathematics Achievement

Tyler A. Womack, Gregory Palardy, and Soojin Park

University of California Riverside, Riverside, CA

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Correspondence concerning this article should be addressed to Soojin Park, Department of Graduate School of Education, University of California Riverside, 1207 Sproul Hall, Riverside, CA, 92521. Email: soojinp@ucr.edu

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Abstract

Mathematics literacy is crucial in many STEM fields, yet Black and Hispanic students are less likely to achieve high math proficiency. While previous literature investigated potential factors to mitigate racial or gendered disparities in mathematics literacy, few studies attended to the conditions under which the causal interpretation of the results obtained can be established. Guided by intersectionality theory and causal decomposition analysis, we examined the degree to which disparities in mathematics literacy (a) exist at the intersection of race and gender and (b) can be reduced by hypothetical interventions that equalize school socioeconomic status (SES) or opportunity to learn (OTL) across groups. We found large racial/ethnic differences in math literacy favoring Asians and whites and much smaller gender differences. We also found that equalizing school SES may reduce disparities for Black and Hispanic males and females; equalizing OTL may reduce disparities for Black and Hispanic males as well as Asian males and females; compared to white males. Our findings suggest that interventions that target specific race–gender groups are required to reduce disparities in math literacy.

Keywords: Intersectionality; Math Literacy; School Socioeconomic Status; Opportunity to Learn; Causal Decomposition Analysis

Inequality of educational opportunity is one of America's most enduring social issues, contributing to racial, gender, and socioeconomic disparities in educational outcomes (Carter & Welner, 2013). Those disparities contribute to gaps in a range of life outcomes related to social mobility such as educational attainment, career opportunities and earnings, wealth, civic participation, and health (Ma et al., 2019). While considerable research has documented educational disparities related to race/ethnicity, gender, or socioeconomic status (SES) (Broer et al., 2019; Gevrek et al., 2020; Henry et al., 2020), limiting the focus to a single aspect of students' social identity disregards how belonging to multiple marginalized groups exacerbates educational inequities (Bullock, 2018). To critically examine issues of educational inequality, *intersectionality* is a framework that has been employed to understand how social categorizations, such as race, gender, and class, intersect to create overlapping and interdependent systems of discrimination or disadvantage within societal institutions (Crenshaw, 1989; Collins, 2015). Research that overlooks intersectionality may distort findings for particular intersectional groups (Bullock, 2018). Therefore, we examine race–gender intersectional disparities in mathematics literacy.

Math is a critical subject for entry into a range of college majors that provide access to high-paying careers in STEM fields. Yet, math-intensive STEM careers remain racialized and gendered, with a large majority of math-intensive fields disproportionately represented by whites, Asians, and men as compared to Blacks, Latino/as, and women (National Science Foundation [NSF], 2017). In regards to racial disparities in mathematics, Black and Hispanic students tend to underperform in high school math standardized tests compared to white and Asian students (National Center for Education Statistics, 2018). Although, on average, there are no significant differences between male and female math scores during middle and high school, women are less likely to pursue math-intensive STEM degrees in college as compared to men (Reardon et al., 2019; Stearns et al., 2020). Examining representation by race and gender reveals larger disparities within math-intensive STEM fields. For instance, Hispanic and Black men constitute only 4% and 3%, respectively, of scientists and engineers in the workforce; and Hispanic and Black women each account for only 2% in these fields (NSF, 2017). To address gaps in math performance and successfully increase the representation of females and racial/ethnic minority students in the STEM workforce, it is essential to examine intersectional disparities.

To reduce intersectional disparities, we consider student opportunity to learn (OTL) and school socioeconomic status (SES) interventions for reducing math disparities. OTL and school SES are among the most robust school-based contributors to high school math achievement and math disparities (Barnard-Brak et al., 2018; Floden, 2002; Palardy et al., 2015; Van Ewijk & Sleegers, 2010). SES is typically a composite measured of parental educational attainment, parental occupational status, and family wealth or income, and has a rich history in sociological and educational research. School SES describes the average SES of students attending a school, which varies across schools due to nonrandom sorting of students into schools. Most of previous studies construct school SES by aggregating student SES at the school level, as our study does (Palardy, Rumberger & Butler, 2015). However, when data sources do not have measures of student SES available, proximal measures such as the percentage of student who receive free and reduced-price lunch or census measures of local neighborhood income, have been used, although some studies have been critical of those alternatives (Harwell & LeBeau, 2010). School SES has been known to have one of the largest effects on educational outcomes among all school-based factors (Benito et al., 2014; Van Ewijk & Sleegers, 2010; Palardy, 2020). Research indicates that

Black and Hispanic students are more likely than white students to attend low SES schools and thus be disproportionately negatively impacted (Jencks & Mayer, 1990; Palardy, 2020; Rumberger & Palardy, 2005). OTL encompasses a variety of educational resources, practices, and conditions that have a direct influence on students' ability to successfully engage with material that aligns with national content standards, especially within STEM education (Bottia et al., 2018). Although OTL encompasses a comprehensive range of educational factors, this study focuses on OTL that focuses on instructional opportunities. Specifically, we define OTL as a measure of "whether or not students have had an opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test" (Husen, 1967a), which was conceptualized by the International Association for the Evaluation of Educational Achievement (IEA). We focus on this instructional OTL since research suggests that it contributes to socioeconomic, racial/ethnic, and gender achievement gaps (Floden, 2002; Husen, 1967a; McGraw et al., 2006; Riegle-Crumb, 2006; Schmidt et al., 2015).

The purpose of this study is to utilize the intersectionality framework to highlight how school-based interventions may reduce math disparities differently by race–gender group. Intersectionality is important in evaluating OTL and school SES interventions because processes through which math disparities occur include differential exposure (i.e., students are exposed to OTL and school SES differently) and differential effects (i.e., students benefit from the interventions differently) based on race–gender. In addition to the substantive contributions of using the intersectional framework, this study makes methodological contributions to the current literature. We use causal decomposition analysis (Jackson & VanderWeele, 2018; Jackson et al., 2016), which allows us to approach the disparity issue with an interventional perspective—for example, what if we could hypothetically intervene to increase Black students' OTL to the level of white students? We outline the assumptions (e.g., no omitted confounding) underlying this approach to maintain a causal interpretation of disparity reduction due to these interventions. In addition, we conduct sensitivity analysis to address possible violations of the no omitted confounding assumption (Park et al., 2023). Using this approach, this study rigorously evaluates the roles of the hypothetical interventions in reducing disparities. The R code used for all analyses is available in the GitHub repository of the corresponding author at XX.

Using a nationally representative sample of U.S. 15-year-olds from the Programme for International Student Assessment (PISA), this study aims to answer the following research questions.

1) To what extent do race–gender intersectional disparities exist in mathematics literacy?
2) To what extent would the disparities be reduced if school SES was the same between marginalized and non-marginalized groups?
3) To what extent would the disparities be reduced if instructional OTL was the same between marginalized and non-marginalized groups?

The second and third research questions require hypothetical interventions of equalizing school SES or instructional OTL between the groups, which we refer to as interventions 1 and 2, respectively.

Background

Intersectionality Theory and Mathematics

Although mathematics has often been portrayed as a politically neutral and objective subject (Bullock, 2018; Skovsmose, 2020), research that addresses diversity issues in U.S. mathematics education has shown how white males have been privileged (Bjorklund-Young & Plasman, 2020; Barnard-Brak et al., 2018; National Science Foundation [NSF], 2017). Yet this research has struggled to adequately confront the intersectional nature of marginalization (Collins & Bilge, 2016; Ireland et al., 2018). Intersectionality provides a framework for understanding the matrix of domination within mathematics education, wherein interlocking systems of privilege and oppression based on identity (e.g., race, class, gender, etc.) are organized through social institutions (Collins, 1990, 2015). The development of intersectionality was largely influenced by the work of Crenshaw (1989, 1991), who critiqued feminist theories and antiracist politics for their tendency to view discrimination as a single-axis framework. By treating race and gender as mutually exclusive categories, discrimination was defined based on the experiences of privileged individuals in terms of their race or sex. Crenshaw argued that Black women were marginalized by feminist and antiracist discourse because it failed to consider their experiences of discrimination at the intersection of race and gender. Since Crenshaw's initial work, the concept of intersectionality has evolved to include other systems of power, such as race, class, gender, sexuality, ethnicity, nation, ability, and age. These categories are no longer viewed as singular and mutually exclusive categories, but as interacting in complex ways that shape social inequalities (Collins, 2015).

Intersectionality explores the experiences of individuals who encounter multiple forms of oppression simultaneously and are often overlooked within mathematics discourse. Bullock (2018) identified this phenomenon in the context of mathematics education, arguing that researchers must critically examine how current political analytical lenses may reinscribe divisions that qualify some individuals and disqualify others in mathematics discourse. For example, in a study on high school students' mathematics motivational beliefs, Hsieh et al. (2021) found that despite Black and Latina/o individuals typically being marginalized in STEM fields, Black and Latina females were more likely to exhibit lower mathematics motivational beliefs than their male peers of the same race. This suggests that while race/ethnicity can lead to marginalization in math, gender can also influence the experiences of these groups in unique ways. Therefore, examining race and gender separately may obscure the experiences of doubly marginalized mathematics students.

Furthermore, the experiences of individuals who belong to both privileged and marginalized groups, such as Asian American women, are also often overlooked in mathematics research. Gibson et al. (2014) revealed that when Asian American women were exposed to mathematical stereotypes related to their race and gender, those who were made aware of the gender stereotype performed worse than those who were exposed to the racial stereotype. Therefore, an intersectional approach not only helps to identify the most vulnerable groups defined by both gender and race/ethnicity, but it also provides the opportunity to analyze whether interventions are effective in reducing mathematics disparities across groups. Specifically, this study aims to investigate if equalizing instructional OTL or school SES can reduce disparities in math literacy at the intersection of race and gender.

Opportunity to Learn

OTL is one of the most important predictors of student achievement, irrespective of a student’s education, income level, or prior academic performance (Barnard-Brak et al., 2018; Schmidt et al., 2015). OTL refers to the range of educational resources, practices, and conditions that directly impact students' capacity to effectively engage with material aligned with national content standards (Bottia et al., 2018). This broad definition includes various elements crucial for student success in STEM education, such as access to rigorous courses (e.g., AP courses), number of credentialed teachers, and availability of high-quality curricula (Schiller et al., 2010; Brown et al., 2019; Xuan et al., 2019). OTL comprises both structural and instructional components (Covay Minor et al., 2015; Urick et al., 2018; Wronowski et al. 2022). Instructional component of OTL pertains to the quality of learning experiences students encounter within their math classes, while structural component of OTL entails the regulation of students' access to math courses through tracking systems (Covay Minor et al., 2015; Urick et al., 2018; Wronowski et al., 2022). Despite the distinction between structural and instructional components of OTL, it is frequently noted that they are closely intertwined, as the track in which students are placed can also impact the quality of the content they receive (Ngo & Velasquez, 2020). It is also worth noting that OTL can have both within-school and between-school effects. The within-school effects of OTL are primarily manifested through structural features such as tracking. In contrast, the between-school effects are due to variation across schools in access to advanced courses and rigorous instruction (Schiller et al., 2010). Among the various elements of OTL, we operationalize it as instructional opportunities, irrespective of whether they stem from tracking, instructional quality, or from within-school or between-school effects.

Mathematics is one of the most commonly tracked subjects in US middle and high schools. When a student is placed in a lower-level math course, their opportunities for advancement can be limited because each math course is viewed as a prerequisite for the next course in the sequence (Ngo & Velasquez, 2020; Riegle-Crumb, 2006). Black and Hispanic students, compared to white and Asian students, are often underrepresented in advanced math courses (de Brey et al., 2019; Leung, et al., 2021). Research shows that Black and Hispanic students who are assigned to lower division courses receive less enriched content and fewer cognitively demanding experiences in their math courses (Barnard-Brak et al., 2018; Wronowski et al., 2022).

When considering OTL through an intersectional lens, it becomes clear that there is significant variation in the experiences of students at the intersection of race/ethnicity and gender. For example, Leyva et al. (2021) conducted a study that looked specifically at the perceptions of Black and Latina/o students in precalculus and calculus classrooms, shedding light on the ways in which gender and race affect the classroom experiences of these students. The authors discovered that Black and Latina females perceived challenging classroom interactions with their teachers through both gendered and racialized perspectives. On the other hand, Black and Latino men focused on their struggle with course content through a racialized lens. Furthermore, Copur-Gencturk et al. (2020) revealed that teachers exhibited biases against Black, Hispanic, and female students regarding their mathematical ability when compared to white males. The study also found that the strongest biases were against Black and Hispanic girls. These studies highlight the double marginalization that women of color may face in mathematics, as they experience discrimination not only based on their race but also their gender.

Therefore, to identify patterns of both marginalization and privilege that exist among specific subgroups, it is crucial to consider the intersection of race and gender within mathematics OTL. The current study builds upon previous research by not only comparing the impact of OTL across races, but also examining how equalizing OTL can decrease performance disparities across different race–gender groups.

School SES

School SES refers to the economic and social background of the students who attend a particular school. Critically, its effect is conceptualized as above and beyond, or controlling for, the students' individual SES (Rumberger & Palardy, 2005) and is typically determined by factors such as family income, education level, and occupation (Van Ewijk & Sleegers, 2010). School

SES can have a significant impact on student academic performance (Benito et al., 2014; Owens & Tom, 2022; Van Ewijk & Sleegers, 2010) and a range of other outcomes, including high school dropout, college enrollment, and the development of social and emotional skills (Palardy, 2020).

School SES can impact student achievement through peer and institutional effects. Peer effects describe when students internalize social norms about mathematics achievement or when peers influence others through disruptive, competitive, or motivational behaviors (Bozick et al., 2010; Goldsmith, 2011; Palardy, 2015, 2019). Institutional effects pertain to how a school's resources, structures, and practices, such as funding, the rigor of available curriculum, and the quality of the teachers, impact student outcomes (Rumberger & Palardy, 2005). Generally, high SES schools tend to offer more resources and opportunities, such as smaller class sizes, experienced teachers, advanced coursework, and extracurricular activities, which can lead to improved academic outcomes. Conversely, low SES schools often have high student to teacher ratios, non-credentialed teachers, and fewer advanced placement courses (Owens, 2018).

Differential school SES could be one plausible explanation for the continued existence of racial/ethnic achievement disparities within the same SES group. Specifically, Black and Latinx students tend to have lower socioeconomic status, and, due to patterns of residential segregation, tend to be concentrated in lower SES neighborhoods and attend lower SES schools as compared to white and Asian students (Owens, 2018; Owens, 2020; Palardy, Rumberger, & Butler, 2015). The intersectionality framework has been applied to investigate school SES effects on mathematics achievement at the intersection of student's racial identity and family SES. For example, Owens and Tom (2022) examined the intersection of racial/ethnic and SES inequalities in mathematics achievement among a sample of California students. Even after controlling for family background, Black and Hispanic students attended low SES schools more frequently than white and Asian students, and these differences in school socioeconomic composition were linked to math achievement disparities within racial/ethnic groups of the same SES. However, when Black and Hispanic students attended schools with similar SES compositions as their white and Asian peers, the math achievement gaps between racial/ethnic groups were reduced (Owens & Tom, 2022). Our study builds upon this research by investigating the potential of reducing disparities in mathematics achievement among students at the intersection of race and gender through equalizing school SES.

Methods

Data

The PISA is an international survey sponsored by the OECD to collect data on student and school performance every three years. The academic subject of emphasis is rotated between mathematics, reading, and science literacy each collection cycle. In 2012, the focus was mathematics literacy. A stratified cluster sampling design was used to collect data from a random sample of 15-year-old students within schools from over 65 countries. Because our interests are in the U.S. population, we limited our analysis to the U.S. sample. We further restricted our sample to white, Black, Hispanic, and Asian students because the “Multiracial” and “Other” categories had small samples with low statistical power. This resulted in a total unweighted sample of 4,597. Sample weights provided within the data were used to obtain a nationally representative sample of students based on race/ethnicity and SES.

PISA 2012 was used for the present study because it includes a vast number of items measuring student’s math perceptions, experiences, attitudes, and mathematics achievement levels that are not included in other data sources. Relatedly, PISA 2012 includes student reports of their knowledge and understanding of specific mathematics topics that can be used to measure instructional OTL. In contrast, most large-scale surveys measure instructional OTL based on teacher reports of content coverage and time spent. Schmidt et al. (2021) argue that student reports of OTL are preferable because when students do not recall specific content, even if it was covered in class, insufficient time may have been provided to learn it.

Measures

Mathematics Literacy

Mathematics literacy is a scaled score developed by PISA consisting of subscales of quantity, uncertainty and data, change and relationship, space and shape, and mathematical processes. This measure quantifies students' ability to analyze, reason, and communicate ideas effectively as they solve mathematical problems in diverse situations (OECD, 2003).

Instructional Opportunities to Learn

Instructional OTL was based on the *index of exposure to formal mathematics*, which was constructed by PISA and is the average of the three following scales (OECD, 2014). *Familiarity with algebra* was the average of three student items rating familiarity with exponential functions, quadratic functions, and linear equations. *Familiarity with geometry* was the average of four student items rating familiarity with vectors, polygons, congruent figures, and cosines. *Familiarity with algebra* and *geometry* was recoded to 0 to 4, with 0 representing "never heard of it" and 4 representing "knew it well." The third scale came from an item that presented two problems, "Solve 2x + 3 = 7" and "Find the volume of a box with sides 3m, 4m, and 5m," and then asked students to rate how often they have encountered these types of problems in mathematics lessons and tests (OECD, 2014). The two items were recoded so that "frequently," "sometimes," and "rarely" equaled 1 and "never" equaled 0 and then averaged together. In our study, the within- and between-school effects are inseparable, and thus the OTL effect may represent a combination of both.

School SES

School SES was taken as the average SES of the students that attended the same school. SES was measured based on PISA's index of economic, social, and cultural status. This index was constructed by PISA using principal component analysis of five parental measures: mother's and father's occupation and years of schooling, family wealth, and household possessions.

Analytical Strategy

We compute the following three estimates to answer our research questions. First, the initial disparity quantifies the gap in mathematics achievement between marginalized and non-marginalized groups. Second, the disparity reduction quantifies the extent to which the initial disparity would be reduced if we hypothetically equalized school SES or OTL across groups. Third, the disparity remaining quantifies the gap left even after intervening on school SES or OTL. Guided by previous literature and a directed acyclic graph (DAG), we control for covariates that may capture omitted variable bias. Also, we use sensitivity analysis that assesses the validity of findings against possible omitted variable bias.

Directed Acyclic Graph

DAG is a transparent tool that encodes causal relationships between variables to better communicate with other investigators. To causally identify the disparity reduction and remaining effects, we must assume there is no omitted variable between math literacy and the intervening variable (school SES or OTL), given the covariates in the respective models. Having no omitted variables is a strong assumption that cannot be tested empirically. Using DAG, we explicitly present our hypothesized causal structure and use it to identify a set of controls needed to correctly estimate the effects of our interest.

[Insert Figure 1 here]

Figure 1 depicts a DAG representing the causal structure between intersectional groups (R) and math literacy (Y). We consider two intervening variables on the pathway from intersectional groups to math literacy: school SES (X) and OTL (M). Let C denote baseline covariates (i.e., home language), L denote the rest of student- and school-level controls, and H denote historical processes such as slavery and Jim Crow, which contribute to differences in various outcomes (Jackson & VanderWeele, 2018). The historical processes are not measured; however, they are shown in DAG to illustrate potential pathways that may explain racial disparities. An arrow from one variable to another indicates that we assume that the first variable causes the other. Three arrows emanating from baseline covariates C indicate that these variables affect all other variables. To eliminate confounding between an intervening variable and the outcome, it is essential to control for all variables identified in the DAG that affect the respective intervening variable and math literacy. For example, when measuring the effect of intervention 1 (school SES), we control for baseline covariates (C) and childhood SES (S). Similarly, to measure the effect of intervention 2 (OTL), we additionally include school SES (X) and intermediate confounders (L) in the analysis.

We assume that racial and gendered disparities in math literacy through school SES (X) arise through the following paths: P1) front-door paths from race and gender to math literacy through school SES (X) (R→X→Y, R→X→M→Y), P2) back-door paths from race and gender to math literacy through history, childhood SES, and school SES (R←H→S→X→Y, R←H→S→X→M→Y), and P3) paths that do not operate through school SES (R→Y, R→

M→Y, R←H→S→M→Y, R←H→S→Y). According to Jackson and VanderWeele (2018), path P2 represents the effect of historical processes, including racism and segregation (Kaufman, 2008). For example, Blacks are more likely to be born into families with low SES and thus live in neighborhoods with low-quality schools. Similar paths could be considered for OTL (M).

Student and School Controls

As shown in our DAG, we hypothesized that home language and student SES confound the relationship between school SES and math literacy. Therefore, we controlled for home language (Muench, Wieczorek & Dressler, 2022) and student SES (Takashiro, 2017; Xuan et al., 2019; Palardy, 2020). Unlike the relationship between OTL and math literacy, we did not assume direct effects of other student- and school-level covariates (such as percent Black, school location, etc.) on school SES. Instead, we consider that these covariates affect school SES indirectly through race–gender intersectional status and student SES of those who attend the school. Therefore, these controls do not qualify as confounders in the school SES–math literacy relationship.

In addition, we hypothesize that baseline covariates, student SES, and the other student- and school-level controls confound the OTL–math literacy relationship. We carefully chose these controls based on previous literature. At the student level, these controls include home language (Urick et al., 2019; Estrada et al., 2020) and student SES (Palardy, 2020; Yang et al., 2018; Wronowski et al., 2022). Grade level and grade repetition (Estrada et al., 2020) were also used as proxy measures for prior achievement because PISA is a cross-sectional study. A set of scales that measure math attitudes and effort (i.e., intentions, subjective norms, work ethic, and study time) was also included because research and theory on planned behavior suggest that they are predictive of math course-taking, which corresponds with OTL (Ajzen, 1991).

At the school level, we controlled for differences among schools that previous research suggests are associated with math literacy and OTL (e.g., Schmidt et al., 2014; Schmidt et al., 2015). Four types of school controls were used, including instructional resources (the proportion of teachers certified; Urick et al., 2018; Wronowski et al., 2022), student composition (percent ELL and percent Black; Estrada et al., 2020; Palardy, Rumberger, & Butler, 2015), structural features (whether the school was located in a city and whether the school was public; Urick et al., 2018), and curricular stratification (whether the school sorts students into curricular tracks or ability grouping, Palardy, 2013; Wronowski et al., 2022). The operational definition of each covariate is shown in Table 1.

[Insert Table 1 here]

Initial Disparity

We estimate initial disparities to address research question 1. Building upon the intercategorical approach utilized in Bauer and Scheim (2019) and informed by intersectionality theory, the combination of self-identified race and gender results in eight intersectional groups: white male, white female, Black male, Black female, Hispanic male, Hispanic female, Asian male, and Asian female. Among these groups, we should first decide which group will be the reference group for comparisons. We use white males as the reference group because white Americans constitute the majority of the students, and males have shown a higher inclination toward pursuing math-intensive STEM degrees in college compared to females (Reardon et al., 2019). The initial disparity is defined as the average difference in math literacy between white males and the respective comparison groups controlling for the baseline covariates. Following VanderWeele and Robinson (2014), causal effects are not specified for socially defined constructs such as race and gender because they are essentially non-modifiable. Instead, we compute the observed disparity between the groups within the same age and home language level. The initial disparity can be obtained by fitting the following model:

$$Y = \alpha_0 + \sum_{r=1}^{7} \alpha_r I(r) + \alpha_c C + \varepsilon_1,$$

where $\sum_{r=1}^{7} I(r)$ for $r \in R$ indicates dummy variables that represent the seven comparison groups, and C is baseline covariates. The term α_r is the initial disparity between the reference group and a given comparison group of $R = r$, within the same home language. In Figure 1, the observed disparity is induced by (or along) paths P1, P2, and P3.

Disparity Reduction/Remaining

Given that one cannot modify social characteristics such as gender and race, identifying modifiable factors is a key feature of intersectionality and educational outcomes. Disparity reductions due to intervening on each factor will be estimated to address research questions 2 and 3. To causally identify these effects, we must assume 1) no omitted confounding between math literacy and the intervening factors, given the covariates in the respective models, 2) consistency, implying that one's observed outcome under the actual value of a variable equals the outcome that would be observed upon intervening to set the variable to that value, and 3) positivity, implying that all comparison groups should have a possibility of experiencing all levels of the intervening variable given covariates and confounders.

For interventions 1 and 2, we use the product-of-coefficients estimator (Jackson & VanderWeele, 2018; Park, Lee & Kang, 2023). We demonstrate each step of the estimator using Intervention 1 (school SES):

1) We fit the outcome model as

$$Y = \beta_0 + \sum_{r=1}^{7} \beta_r I(r) + \beta_s S + \beta_x X + \beta_c C + \sum_{r=1}^{7} \beta_{rx} I(r) \cdot X + \varepsilon_2,$$

where $\sum_{r=1}^{7} I(r)$ for $r \in R$ indicates dummy variables that represent the seven comparison groups, S is student SES, X is school SES, and C is baseline covariates. Here we include interaction effects between the intersectional groups and school SES $(I(r) \cdot X)$. As a result, $\beta_x + \beta_{rx}$ is the effect of school SES for comparison group of $R = r$ after controlling for student SES and home language. In Figure 1, $\widehat{\beta_x} + \widehat{\beta_{rx}}$ provides estimates for the path from X to Y after controlling for previously measured confounders.

2) We fit a mediator model as $X = \gamma_0 + \sum_{r=1}^{7} \gamma_r I(R = r) + \gamma_c C + \varepsilon_3$,

where $\sum_{r=1}^{7} I(r)$ for $r \in R$ indicates dummy variables that represent the seven comparison groups, and C is baseline covariates. The term γ_r is the difference in school SES between the reference group and a given comparison group of $R = r$, within the same home language. In Figure 1, $\widehat{\gamma_r}$ provides estimates for the paths from R to M after controlling for baseline covariates C.

3) Using the parameter estimates obtained from the previous steps, we can estimate the disparity reduction for a comparison group of $R = r$ as $\widehat{\gamma_r} \times (\widehat{\beta_x} + \widehat{\beta_{rx}})$, and the disparity remaining for a comparison group of $R = r$ as $\widehat{\alpha_r} - \widehat{\gamma_r} \times (\widehat{\beta_x} + \widehat{\beta_{rx}})$. In Figure 1, disparity reduction due to Intervention 1 is represented by the associations transmitted along paths P1 and P2, while disparity remaining is represented by the associations transmitted along path P3.

Note that we did not control for student SES (S) in step 2. This decision was intentional, as our goal is to include the path from race–gender to school SES through student SES when estimating the disparity in school SES between race–gender groups. This choice was made because it is important to examine the disparity in school SES across different levels of student SES, rather than focusing on a specific level of student SES.

For Intervention 2, the analytical steps are the same as for Intervention 1, except that instead of using school SES to fit the mediator and outcome models, OTL is used. The rate of missing data for student- and school-level variables was mostly very low (0–7%) but was 33% on the OTL items. We imputed missing data using predictive mean matching (Little, 1988). We used clustered bootstrapping to estimate standard errors that account for the data structure in which students are nested within each school. The procedure and its rationale are described in Huang (2018).

Sensitivity Analysis

Another advantage of using the causal framework is that we use sensitivity analysis to determine when our findings would be invalidated due to omitted variables. Sensitivity analysis aims to examine how much the disparity reduction estimate would change if omitted variables were measured and controlled. To do this, we adopt a sensitivity analysis technique proposed by Park et al. (2023). The technique requires two sensitivity parameters: 1) the partial R^2 of an omitted variable with the outcome, after controlling for intersectional status, the corresponding intervening factor, and confounders, and 2) the partial R^2 of an omitted variable with the intervening factor, after controlling for intersectional status and baseline covariates. These two sensitivity parameters represent the associations between the omitted variable and the intervening factor as well as outcome variables after controlling for the relevant covariates. If a potential omitted variable has a strong association with the intervening variable (school SES or OTL) and the outcome (math literacy), the disparity reduction estimates could become closer to zero, nonsignificant, or even reverse signs.

The idea is to find the combinations of two sensitivity parameters that make the disparity reduction estimates zero or change the significance of the estimates at the 95% confidence level.

For example, we consider the result as sensitive to violations of the no omitted confounding assumption if the disparity reduction loses significance, even in the presence of an omitted variable that explains a small proportion of the variance in the mediator and outcome variables (e.g., $R^2 < 0.05$). In contrast, we consider the result robust to violations of the assumption if the disparity reduction remains significant with relatively large R^2 values (e.g., $R^2 > 0.10$). After accounting for existing confounders, it is uncommon for omitted variables to explain, for example, 10% of the variance of the intervening variable and the outcome. However, determining the specific threshold for R^2 is challenging as it depends on the context of individual studies. We adopt a strategy of using the strongest existing covariate as a threshold, which will be explained shortly. Decomposition and sensitivity analyses were carried out using the '*causal.decomp*' R package (Kang & Park, 2022).

Results

Descriptive Statistics

Table 2 shows the means and standard deviations for the outcome, intervening variables, and covariates by race–gender group. Large racial/ethnic differences in math literacy were found, with Asian Americans scoring the highest, on average, 0.76 standard deviations (SD) above the overall mean, followed by whites (0.27 SD), Hispanics (-0.30 SD), and Blacks (-0.67 SD). We also found race–gender intersectional effects. Black females scored higher than Black males (difference = 0.13 SD), Asian females and males scored equivalently, and Hispanic and white males scored higher than their female counterparts (difference for Hispanics = 0.10 SD and difference for whites = 0.14 SD). The extensive racial/ethnic disparities and race–gender intersectional effects are consistent with previous studies (McGraw et al., 2006).

[Insert Table 2 here]

Large racial/ethnic differences in school SES were found, with whites and Asians attending schools with higher SES than the average (whites: 0.30 SD and Asians: 0.32 SD), followed by Blacks (-0.25 SD), and Hispanics (-0.68 SD). Gender differences in school SES are present but negligible. We also found that the level of instructional OTL varies significantly by race/ethnicity and gender. Asians have the highest OTL (0.69 SD), followed by whites (0.07 SD), Hispanics (-0.18 SD), and Blacks (-0.20 SD). Within the same race/ethnicity group, females tend to have higher OTL than males, particularly among Blacks, with Blank females having 0.25 SD greater OTL than Black males.

Disparities Before and After the Interventions

The second row of Table 3 reports the standardized estimates for the initial disparity for each comparison group (compared with white males) after controlling for home languages. We observe the most significant disparity for Black males (-1.10 SD) and females (-1.00 SD), followed by Hispanic females (-0.64 SD) and males (-0.48 SD), and white females (-0.17 SD). These results indicate that these groups achieve lower math scores than white males. In contrast, the initial disparities are positive for Asian males (0.57 SD) and Asian females (0.38 SD), indicating that these groups score higher in math achievement than white males.

[Insert Table 3 here]

The next two blocks present standardized estimates for disparity reductions due to each intervention and disparity remaining after the intervention. Intervention 1 (school SES) significantly reduces the initial disparities for Blacks and Hispanics, although the magnitude of reduction varies by group. The reduction is the largest for Hispanic females (26.6%), followed by Hispanic males (25.0%), Black females (15.0%), and Black males (7.3%). The results indicate that the initial disparities may be reduced by 7.3% (Black males) to 26.6% (Hispanic females) if we set school SES equal between groups among those with the same home language.

Intervention 2 (instructional OTL) significantly reduces the initial disparity for Black and Hispanic males, as well as Asian males and females. Specifically, the estimated reduction for Hispanic males and Black males is 18.8% and 10.0%, respectively. For Asian males and females, the disparity reduction is 57.9% and 124.0%, respectively. These results indicate that if we increase the OTL of Black and Hispanic males to the level of white males with the same home languages, the initial disparities in favor of white males may reduce by 18.8% and 10.0%, respectively. Conversely, if we equalize the instructional OTL between white males and Asian males with the same home languages, the initial disparities in favor of Asian males may reduce by 57.9%. Moreover, if we equalize the instructional OTL between white males and Asian females, the disparity, currently in favor of Asian females, would be reversed, potentially resulting in White males outperforming Asian females.

Although this is not our central interest, we present the degree to which each intervening variable is associated with math literacy for each group in Table 3 (see row labeled "Intervention Effects" in each block). This association is drawn from the outcome model in estimation step 1 (e.g., $\widehat{\beta_x} + \widehat{\beta_{rx}}$ for school SES). School SES is positively associated with math literacy for all groups, although the association varies considerably by group. The association is the largest for Black females (0.28 SD), Asian females (0.22 SD), and Hispanic females (0.20 SD), suggesting that Black, Asian, and Hispanic females benefit more than their male counterparts from attending schools with high SES, given that the conditional independence assumption is met. Instructional OTL is also positively associated with math literacy for all groups, and the magnitude of the association is larger than the association with school SES. The association with OTL varies by race/ethnicity, with Asian females benefitting the most from high OTL (0.65 SD), followed by whites (0.44 – 0.46 SD), and Blacks and Hispanics (0.33 – 0.38 SD).

Sensitivity Analysis

While we carefully chose covariates based on previous literature and the DAG to capture omitted variable bias, omitted confounders could still exist. Therefore, we apply a sensitivity analysis to assess possible violations of this the no omitted confounding assumption. Figures 2 and 3 show the ranges of disparity reduction estimates given the sensitivity parameter combinations (described above).

[Insert Figures 2 and 3 here]

We assess whether the disparity reduction estimates from Table 3 are still valid even when an omitted variable exists as strong as student SES. We use student SES as our reference value because it is widely considered the most robust predictor of various education outcomes, including math achievement (Sirin, 2005). Also, relatively few educational factors explain math literacy more than student SES.

Figures 2A–2D show the change of disparity reduction estimates due to Intervention 1 given two sensitivity parameters. Refer to the footnote for instructions on how to read and interpret the figures. The figures indicate that disparity reduction would no longer be significant if unobserved confounder U explains both school SES and math literacy by 1.0% (Hispanic males) – 5.2% (Hispanic females). The strongest existing covariate, student SES, explains the outcome by 6.4%, after controlling for other confounders. Therefore, if an unobserved confounder is as strong as student SES, the disparity reduction estimates for Black males and females and Hispanic males and females would no longer be significant.

Figures 3A–3D indicate the change of disparity reduction estimates due to Intervention 2 given two sensitivity parameters. The figures indicate that disparity reduction would no longer be significant if unobserved confounder U explains both OTL and math literacy by 9.5% (Black males) – 17.4% (Asian males). Given the partial R^2 values of existing covariates with math literacy, these confounding amounts are large and unlikely. The strongest existing covariate, student SES, explains the outcome by 1.7%, after controlling for other confounders.

Overall, given consistency and positivity, these results suggest that causal interpretations of significant disparity reductions due to Intervention 2 are warranted. In contrast, causal interpretations of significant disparity reductions due to Intervention 1 may not be warranted if unobserved confounders are as strong as student SES. However, it is important to note that consistency is controversial in the context of our study. As a result, the disparity reduction due to an actual intervention may be smaller than estimated, and we discuss this aspect further in the limitation section.

Discussion

Interventions for Reducing Intersectional Disparities

Our study utilized an intersectional approach to investigate the prevalent disparities in mathematics achievement among students at the intersection of race and gender. Furthermore, we explored potential interventions aimed at reducing these gaps and promoting greater equity in mathematics education. Specifically, we explored two hypothetical school interventions to assess their potential to reduce intersectional disparities in math literacy: 1) equalizing school SES, and 2) equalizing instructional OTL between marginalized and non-marginalized groups.

We assessed these interventional effects using causal decomposition and sensitivity analyses under the causal inference framework. This framework offered the following advantages. First, we chose covariates based on a DAG and previous literature. While selecting the right set of covariates is crucial for causal inferences, a conventional approach is often based on selecting covariates from previous literature without clearly presenting the causal structure of variables. Using a DAG and previous literature, we clearly showed the causal relationships among the variables we hypothesized and provided a rationale for choosing covariates for each intervention. Second, we used causal decomposition analysis that approaches disparities issues from an interventional perspective—i.e., what if we hypothetically intervene to equalize the distribution of mediators between marginalized and nonmarginalized groups? The results based on the causal decomposition method indicated that both interventions can potentially reduce mathematics literacy disparities. Third, we used sensitivity analysis to validate our findings from causal decomposition analysis against potential omitted variable bias. Our sensitivity analysis indicated that the effects of Intervention 2 for Black and Hispanic males as well as Asian males and females were robust to omitted variable bias even if we have omitted confounders as strong as student SES.

Who Benefits from the Interventions and Why?

The results for Intervention 1 showed that equalizing school SES across intersectional groups may significantly reduce math literacy disparities for Blacks (-.12 SD for males and -.15 SD for females) and Hispanics (-.12 SD for males and -.17 SD for females). The descriptive statistics in Table 2 provide further insight into these findings. Black and Hispanic adolescents attended schools with SESs that, on average, were 0.6 to 1.0 SD lower than the schools white and Asian students attended. These findings were consistent with previous literature that noted school SES segregation impacted Black and Hispanic students because they were far more likely to attend low SES schools (Owens, 2018; Owens, 2020; Palardy, Rumberger, & Butler, 2015).

Moreover, in previous research, Owens and Tom (2022) utilized the intersectionality framework to examine the impact of both student race/ethnicity and SES on math achievement trajectories. Their findings showed that when Black and Hispanic students attended schools with similar SES to their same-SES white and Asian counterparts, there were smaller racial/ethnic disparities in math achievement. Our study offers a novel perspective by demonstrating that equalizing school SES may reduce the initial disparities for Black and Hispanic males and females with white males.

This study further demonstrated that analyzing only at the level of race would fail to account for the differential benefits received by male and female adolescents of the same race from intervening on instructional OTL. Specifically, Intervention 2 had significant reductions for Black males (-.11 SD) and Hispanic males (-.09 SD), while the reductions were not significant for Black and Hispanic females. As shown in Table 2, Black and Hispanic males exhibit OTL levels 0.36 and 0.23 SD lower than white males, respectively. In contrast, the gap with white males was smaller for Black females and Hispanic females, which were 0.11 and 0.20 SD, respectively. Notably, the level of OTL did not significantly differ between Hispanic males and females, while Black females demonstrate a significantly higher OTL than Black males. This finding was consistent with previous research showing Black females tended to progress further in the math course-taking sequences than Black males (Riegle-Crumb, 2006).

Several studies considered instructional OTL a mediating mechanism that explained the relationship between SES and math literacy (Schmidt et al., 2015; Kang & Cogan, 2022). These findings implied that socioeconomically advantaged students tended to have more exposure to math content, and in turn, this led to high proficiency in math literacy. Our study additionally found that instructional OTL was an important mechanism explaining racial and gendered disparities in math literacy. Specifically, our study suggests that the substantial amount of disparities for Black males and Hispanic males may be reduced if we increase their exposure to math content to the level of white males.

The question that arises is why instructional OTL interventions led to a significant reduction in disparities for Black and Hispanic males, but not for females. Persistent racial and gender disparities in math education present an ongoing challenge to maintaining positive attitudes towards math for Black and Hispanic girls. Despite their proficiency and self-efficacy in math, young Black and Hispanic girls often exhibit lower levels of math motivation and identity (e.g., the view of oneself as a mathematical person) compared to their male counterparts (Hsieh et al., 2021; Young & Cunningham, 2021). As such, it is possible that interventions aimed at improving math motivation and identity may be a critical component, along with OTL, for narrowing the mathematics disparities for these groups.

Mean Effects for Each Intervention

While the interventions varied in their effectiveness for reducing intersectional disparities in math literacy, all interventions were strongly associated with math literacy. The average effect size for interventions 1 and 2 were 0.20 and 0.43 SDs (see the first row of each intervention in Table 3). Each of those effects is large (i.e., Effect Size > 0.2) by educational intervention standards (Kraft, 2020). This suggests that besides reducing disparities, these interventions may have the potential for raising overall math literacy.

Limitations

In this study, we employed an intersectional approach to examine the racial and gendered disparities in mathematics performance and explored how interventions could address disparities in math achievement. However, our approach was constrained by pre-established categories of race and gender within the PISA dataset, which prevented us from capturing variation within specific racial and ethnic groups. To fully understand how a student's identities may impact their experiences in mathematics institutions, we need to consider not only their racial and ethnic identity, but also other factors represented in students' identities such as immigration history, assimilation, and language (Bullock, 2018). Future research should address these limitations by exploring how additional aspects of student identity, beyond race and gender, contribute to disparities in mathematics performance.

The OTL measure used in this study was based on student perceptions. It captured both the quality of instruction and the amount of time provided for students to learn math concepts. Yet, it is difficult to disentangle students' exposure to formal mathematics from their potential recall bias, as students sometimes forget what they were taught. Recall biases may result in measurement error in the OTL variable, which may affect the strength of its association with math literacy.

In addition, causal decomposition analysis is based on consistency—e.g., one's math score under the actual value of school SES equals the outcome that would be observed upon intervening to set school SES to that value. For example, Intervention 1 may involve relocating students between schools to equalize school SES. In geographic areas where poverty is widespread, the transportation cost of moving students can be exorbitant, and the time cost to students can be counterproductive. Consequently, consistency would be violated due to the costs incurred in equalizing school SES, diminishing the benefit of this intervention. Therefore, the interventions' actual effects will likely be smaller than indicated by this study's results. Nonetheless, by simulating hypothetical interventions in observational data, this study can help inform the design of interventions aimed at reducing disparities in mathematics literacy.

Conclusions

This study contributes to the existing literature by investigating how interventions targeting the reduction of mathematics disparities may differently benefit groups based on their race and gender. Specifically, race–gender intersectional disparities were observed, showing that Black females scored higher in math literacy than Black males, while Hispanic and white males outscored their female counterparts. Our results suggest that interventions equalizing school SES can potentially reduce disparities for Black and Hispanic males and females. However, the effectiveness of such interventions may diminish if unobserved confounders, as influential as student SES, are present. More rigorous investigations that use experimental designs are warranted to investigate whether intervening to equalize school SES can reduce mathematics disparities across different race–gender groups.

Our findings also suggest that interventions to equalize OTL may reduce disparities for Black and Hispanic males as well as Asian males and females compared to white males. As such, our results reinforce the notion that students encounter an opportunity gap rather than an achievement gap (Wronowski et al., 2022; Urick et al., 2019). School administrators and policymakers should offer instructional leadership that empowers teachers to enhance OTL for all students. Enhancing OTL can involve optimizing math lesson time, implementing effective math tasks, refining teachers' mathematical teaching strategies, and fostering meaningful math discussions among students (Walkowiak et al., 2017). Moreover, integrating culturally relevant pedagogy that draws on students' backgrounds is important in enabling the application of mathematical concepts to real-world scenarios (Brown et al., 2019).

Finally, our study underscores the significance of examining mathematical disparities through an intersectionality perspective, since interventions will be more effective when targeted to specific race–gender groups. Specifically, intervening on OTL was less beneficial for Black and Hispanic females, highlighting the necessity to explore additional factors in interventions for these groups. Research suggests that enhancing their sense of mathematics identity, such as viewing themselves as mathematical individuals, in conjunction with addressing OTL, may be more effective in closing gaps for these groups (Hsieh et al., 2021; Young & Cunningham, 2021). Future research should further investigate and develop targeted interventions to effectively reduce mathematical disparities for Black and Hispanic females.

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Table 1. *Variable Labels and Descriptions*

Name	Description (PISA name)
Outcome (Y)	
Math Literacy	Math achievement constructed from 5 plausible values (PV1MATH-PV5MATH)
Equity Interventions	
School SES	School mean of student SES (mean of ESCS)
OTL	Opportunity to learn formal math (constructed; see methods section)
Student Covariates	
Student SES	Index of economic, social and cultural status (ESCS)
Grade Level	Grade level (ST01Q01)
Spanish	Spanish is home language (SPANISH)
Other	Other than English or Spanish is home language (OTHER)
Repeat	Repeated a grade level (REPEAT)
Homework Time	Out-of-school study time (OUTHOURS)
Math Work Ethic	Mathematics Work Ethic (MATWKETH composite)
Subjective Norms	Subjective norms in mathematics (SUBNORM composite)
Math Intentions	Mathematics intentions (MATINTFC composite)
School Covariates	
Math S/T Ratio	Math teacher-student ratio (SMRATIO)
Teaching shortage	Math teacher shortage (SC14Q02)
Proportion Certified	Proportion of teachers certified (PROPCERT)
Public	Sector of school (SC01Q01 recoded to 1=public, 0=private)
City	School Location (SC03Q01=4 or 5)
Ability	Ability grouping: Similar content/different difficulty (SC15Q01= 2 or 3)
Tracking	Ability grouping: Different content/different difficulty (SC15Q02= 2 or 3)
% ELLs	Percent of students at school that are ELL (constructed)
% Black	Percent of students at school that are Black (constructed)

[1] MATWKETH was derived from student responses on nine Likert items (ST46Q01-09) concerning their level of agreement with the following statements: 1) I have my homework finished in time for mathematics class; 2) I work hard on my mathematics homework; 3) I am prepared for my mathematics exams; 4) I study hard for mathematics quizzes; 5) I keep studying until I understand the mathematics material; 6) I pay attention in mathematics class; 7) I listen in mathematics class; 8) I avoid distractions when I am studying mathematics; 9) I keep my mathematics work well organized.

[2] SUBNORM was constructed using student responses on six Likert items (ST35Q01-06) regarding thinking about how people important to them view mathematics: 1) Most of my friends do well in mathematics; 2) most of my friends work hard at mathematics; 3) my friends enjoy taking mathematics tests; 4) my parents believe it's important for me to study mathematics; 5) my parents believe that mathematics is important for my career; 6) my parents like mathematics.

[3] MATINTFC was constructed from 5 forced responses items (ST48Q01-05) asking students to choose, for each pair of the following statements, the item that best described them: 1) I intend to take additional mathematics courses after school finishes vs. I intend to take additional courses after school finishes; 2) I plan on majoring in a subject in that requires mathematics skills vs. I plan on majoring in a subject in that requires science skills; 3) I am willing to study harder in my mathematics classes than is required vs. I am willing to study harder in my classes than is required; 4) I plan on as many mathematics classes as I can during my education vs. I plan on as many science classes as I can during my education; 5) I am planning on pursuing a career that involves a lot of mathematics vs. I am planning on pursuing a career that involves a lot of science.

Table 2. *Description Statistics of Variables by Race–Gender Intersectionality Group, Mean (SD) or %*

	Total	White		Black		Hispanic		Asian	
		Male	Female	Male	Female	Male	Female	Male	Female
Sample Size	4597	1276	1277	346	295	593	583	113	114
Outcome									
Math Literacy	0.00	0.34	0.20	-0.74	-0.61	-0.25	-0.35	0.75	0.78
(standardized)	(1.00)	(0.93)	(0.88)	(0.84)	(0.87)	(0.92)	(0.84)	(0.92)	(1.05)
Intervening variables									
School SES	0.00	0.30	0.31	-0.27	-0.22	-0.66	-0.71	0.39	0.26
(standardized)	(1.00)	(0.80)	(0.83)	(0.90)	(0.87)	(1.06)	(1.07)	(1.13)	(1.04)
OTL	0.00	0.03	0.11	-0.33	-0.08	-0.20	-0.17	0.65	0.73
(standardized)	(1.00)	(1.00)	(0.94)	(1.03)	(0.90)	(1.04)	(0.98)	(0.92)	(0.86)
Student Variables									
Student SES	0.17	0.27	0.31	-0.10	-0.11	-0.59	-0.71	0.04	0.30
	(0.98)	(0.85)	(0.89)	(0.87)	(0.84)	(1.03)	(1.01)	(1.22)	(0.95)
Grade	10.05	10.02	10.08	9.90	10.13	10.02	10.06	10.10	10.36
	(0.54)	(0.49)	(0.46)	(0.60)	(0.57)	(0.65)	(0.57)	(0.65)	(0.55)
Spanish	0.11	0.00	0.00	0.01	0.00	0.40	0.44	0.00	0.00
	(0.31)	(0.03)	(0.06)	(0.08)	(0.00)	(0.49)	(0.50)	(0.00)	(0.00)
Other	0.03	0.02	0.01	0.01	0.03	0.00	0.00	0.39	0.35
	(0.17)	(0.14)	(0.10)	(0.12)	(0.18)	(0.09)	(0.04)	(0.49)	(0.48)
Repeat	0.00	-0.05	-0.13	0.33	0.04	0.22	0.05	-0.19	-0.28
	(1.00)	(0.94)	(0.81)	(1.30)	(1.05)	(1.21)	(1.05)	(0.70)	(0.50)
Homework	10.5	8.6	11.2	10.8	11.2	9.7	10.7	13.4	18.6
	(9.7)	(8.4)	(8.2)	(16.3)	(9.5)	(8.9)	(9.0)	(10.8)	(15.5)
Math Work Ethic	0.00	-.11	0.07	0.08	0.15	-0.12	-0.06	0.34	0.40
(standardized)	(1.00)	(1.01)	(0.98)	(1.09)	(0.91)	(0.97)	(0.95)	(1.12)	(0.99)
Subjective Norms	0.00	-0.03	-0.11	0.29	-.06	-.00	-.04	0.66	0.48
(standardized)	(1.00)	(1.01)	(0.95)	(1.12)	(0.90)	(0.97)	(0.98)	(1.04)	(0.89)
Math Intentions	0.00	0.10	-0.22	0.19	-0.14	0.25	-0.04	0.21	-0.02
(standardized)	(1.00)	(1.02)	(0.97)	(0.96)	(0.98)	(1.00)	(0.98)	(0.97)	(0.89)
School Variable									
Math S/T Ratio	120.7	120.0	116.4	117.0	118.7	129.4	125.9	125.1	120.4
	(38.2)	(39.5)	(37.9)	(29.0)	(32.1)	(41.6)	(39.6)	(32.9)	(36.1)
Teacher Shortage	0.00	-0.09	-0.07	0.20	0.09	0.16	0.09	-0.08	-0.17
(standardized)	(1.00)	(0.93)	(0.94)	(0.98)	(0.99)	(1.14)	(1.14)	(0.85)	(0.84)

Certified Teachers	94.9 (13.1)	95.0 (14.3)	94.0() (15.9)	90.5 (14.7)	96.4 (8.4)	96.4 (9.5)	96.7 (8.4)	97.1 (8.7)	96.7 (8.5)
% Public	91.1 (28.5)	92.2 (26.9)	88.8 (31.6)	78.4 (41.2)	90.3 (29.6)	96.8 (17.6)	98.6 (11.9)	87.9 (32.6)	83.0 (27.7)
% City	39.0 (48.8)	22.7 (41.9)	25.9 (43.8)	66.5 (47.3)	66.4 (47.3)	56.0 (49.7)	55.2 (49.8)	46.3 (50.1)	48.1 (50.2)
% Tracking	87.6 (32.9)	88.5 (31.9)	87.1 (33.5)	88.3 (32.2)	83.7 (37.0)	86.3 (34.4)	90.0 (30.0)	86.8 (34.0)	88.7 (31.8)
% Ability	80.5 (39.6)	76.9 (42.1)	75.5 (43.0)	87.4 (33.2)	81.3 (39.2)	86.5 (34.2)	87.9 (32.7)	84.1 (36.7)	86.8 (34.0)
% ELLs	16.9 (20.9)	9.2 (13.4)	9.6 (14.0)	15.9 (20.8)	19.2 (22.6)	33.1 (24.4)	31.8 (24.2)	23.3 (24.2)	25.8 (25.2)

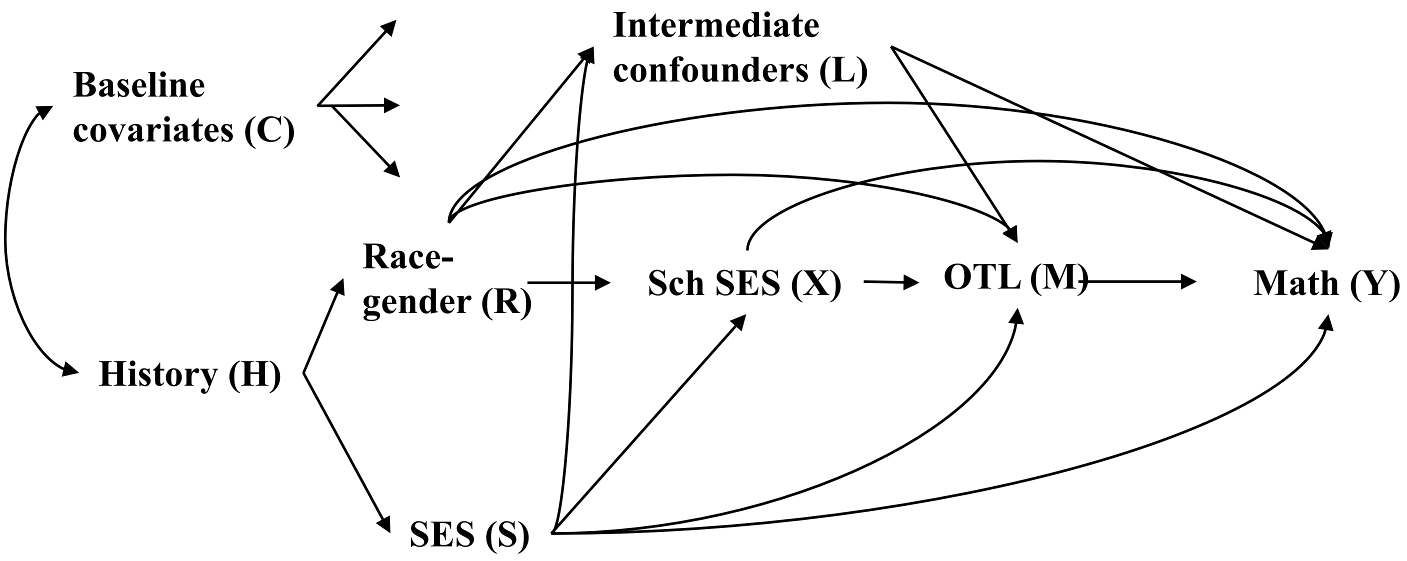
Table 3.

Estimates of Initial Disparity, Intervention Effects, Disparity Remaining, and Disparity Reductions (Confidence Interval) in Mathematics Literacy under two Hypothetical Interventions.

	Comparison Group	White Males	White Female	Black Males	Black Females	Hispanic Males	Hispanic Females	Asian Males	Asian Females
	Initial Disparity	---	-.17 (-.24, -.10)	-1.10 (-1.23,-.96)	-1.00 (-1.18,-.82)	-.48 (-.62, -.34)	-.64 (-.77, -.51)	.57 (.35, .79)	.38 (.10, .65)
Intervention 1: School SES	Intervention Effect	.18***	.13***	.12*	.28***	.13***	.20***	.14†	.22**
	Disparity Remaining	---	-.17 (-.25, -.10)	-1.02 (-1.16,-.84)	-.85 (-1.02,-.66)	-.37 (-.50, -.24)	-.48 (-.62, -.35)	.48 (.31, .68)	.41 (.18, .73)
	Disparity Reduction	---	.00 (-.01, .02)	-.15 (-.20, -.02)	-.15 (-.27, -.06)	-.12 (-.21, -.03)	-.17 (-.23, -.07)	.08 (-.04, .21)	-.04 (-.26, .08)
	% Reduction	---	-	7.3	15.0	25.0	26.6	-	-
Intervention 2: OTL	Intervention Effect	.46***	.44***	.33***	.38***	.35***	.37***	.40***	.65***
	Disparity Remaining	---	-.21 (-.25, -.12)	-.99 (-1.12,-.83)	-.97 (-1.17,-.86)	-.39 (-.52, -.28)	-.58 (-.68, -.46)	.24 (.06, .45)	-.10 (-.22, .23)
	Disparity Reduction	---	.04 (-.03, .06)	-.11 (-.19, -.06)	-.04 (-.08, .09)	-.09 (-.16, -.02)	-.06 (-.15, .00)	.33 (.15, .50)	.47 (.16, .61)
	% Reduction	---	-	10.0	-	18.8	-	57.9	124.0

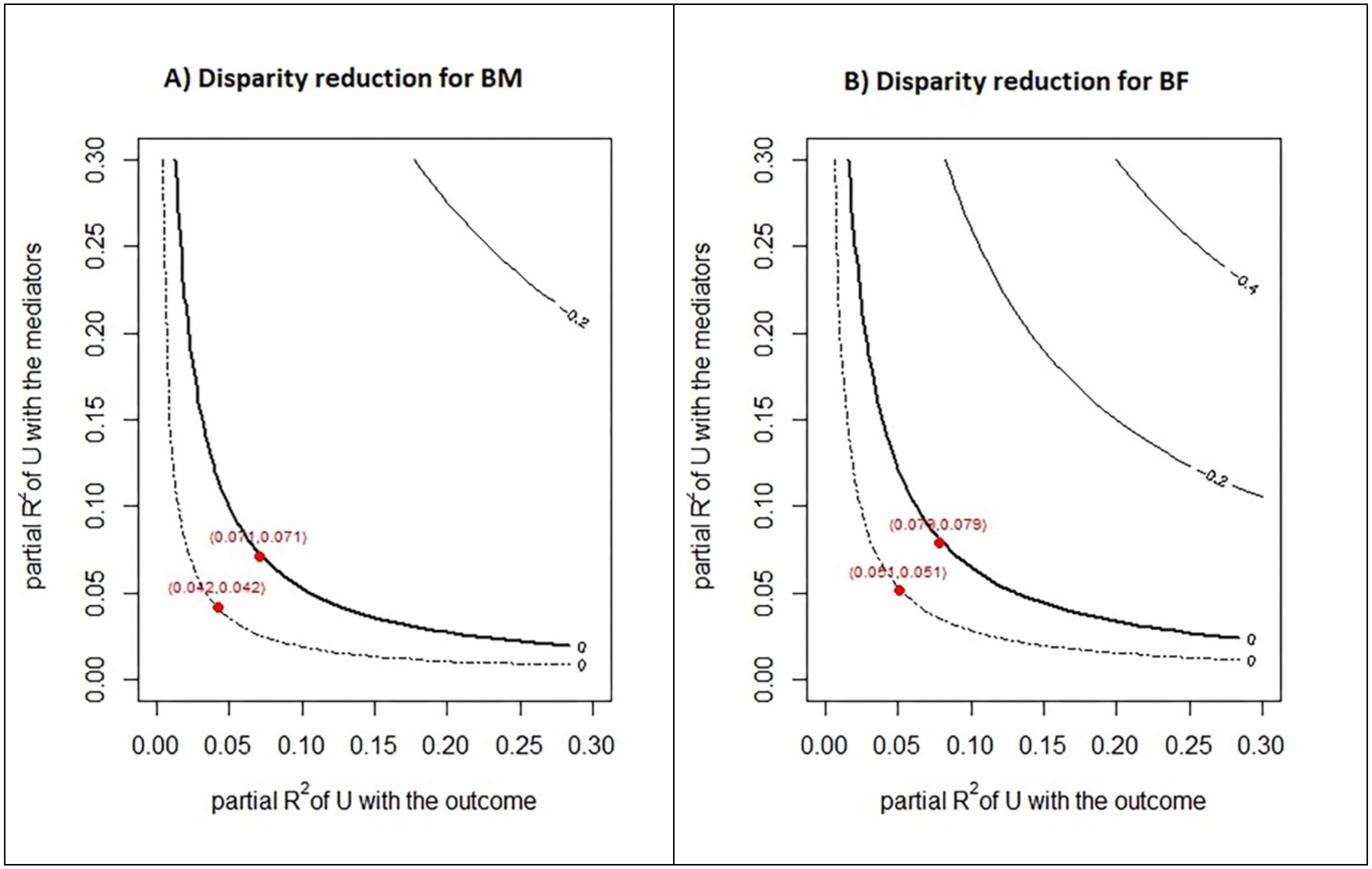
$^{\dagger}p < 0.10$; $^{*}p < 0.05$; $^{**}p < 0.01$; $^{***}p < 0$

Figure 1. *Directed Acyclic Graph of Relationships Between Race–Gender Intersectional Status, Mathematics Achievement, and Potential Mediators.*



Note. 1) Baseline covariates include home languages. 2) Sch SES represents school SES and Math represents math literacy.

Figure 2. *Sensitivity Analysis for Intervention 1*[1].



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[1] BM: Black male, BF: Black female, HM: Hispanic male, and HF: Hispanic female.
The omitted variable is denoted as U. One sensitivity parameter is presented on the X-axis, indicating the association between U and math literacy. Another sensitivity parameter is presented on the Y-axis, indicating the association between U with the mediator (school SES or OTL). Both sensitivity parameters are represented as partial R^2 values. The solid line shows the combinations of the two sensitivity parameters that would result in zero disparity reduction. The dashed line shows the points where the 95% confidence intervals cover zero. The region below the dashed line is where the estimates are still significant despite omitted confounding. The region above the dashed line is when the significance of the estimate changes to non-significance due to omitted confounding. The dots in each figure represent the amount of confounding that will result in zero disparity reduction or change the significance of the estimate to non-significance, given the equal amount of confounding between two sensitivity parameters.

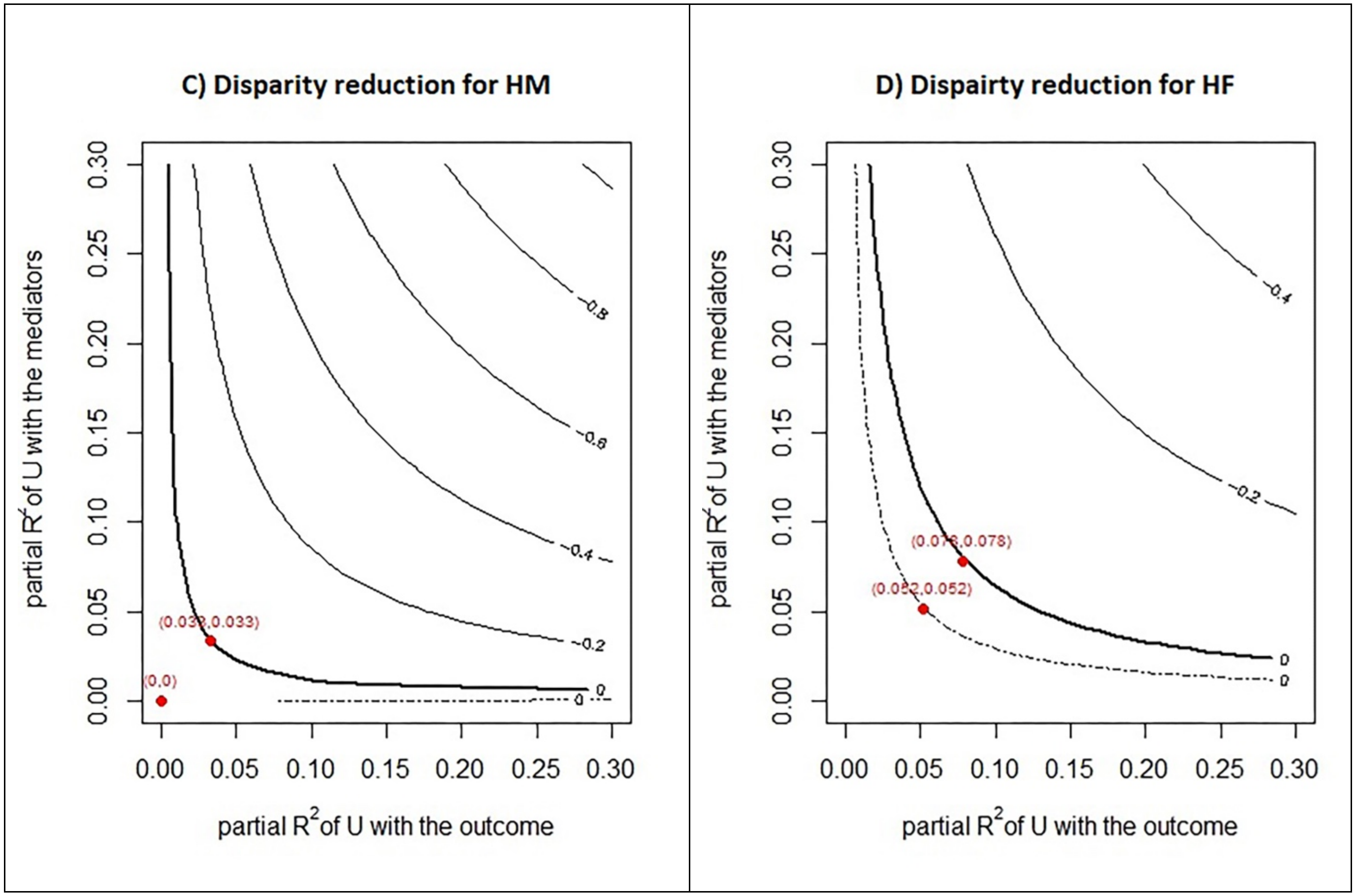
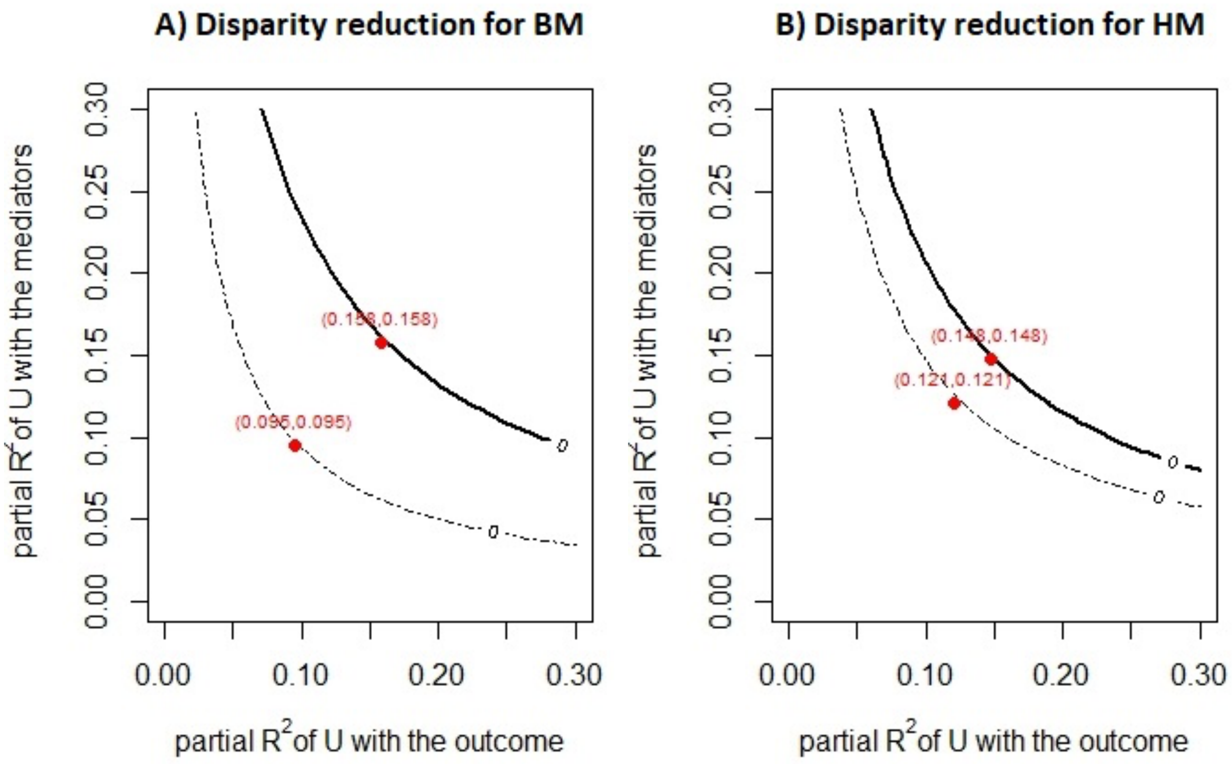

C) Disparity reduction for HM
partial R^2 of U with the mediators
0.00
0.05
0.10
0.15
0.20
0.25
0.30
partial R^2 of U with the outcome
0.00
0.05
0.10
0.15
0.20
0.25
0.30
-0.8
-0.6
-0.4
-0.2
0
0
(0.033,0.033)
(0,0)
D) Dispairty reduction for HF
partial R^2 of U with the mediators
0.00
0.05
0.10
0.15
0.20
0.25
0.30
partial R^2 of U with the outcome
0.00
0.05
0.10
0.15
0.20
0.25
0.30
-0.4
-0.2
0
0
(0.078,0.078)
(0.052,0.052)

Figure 3. *Sensitivity Analysis for Intervention 2*[2].



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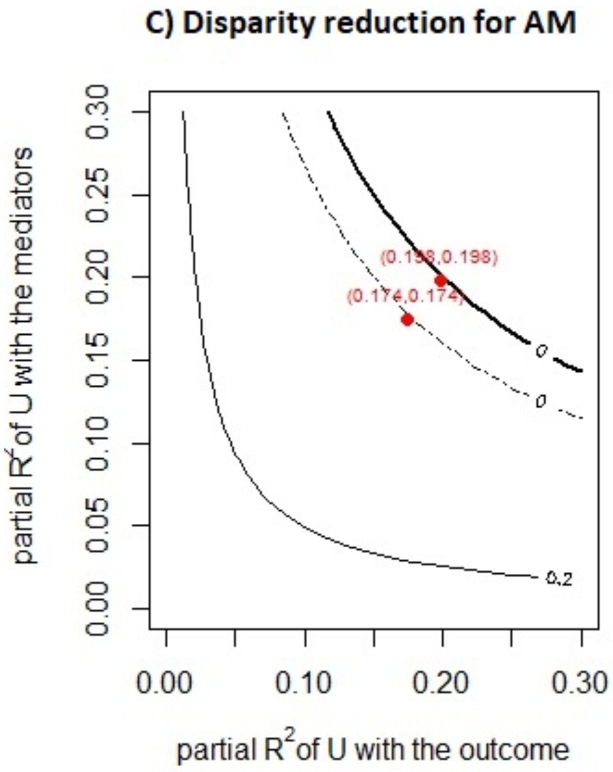
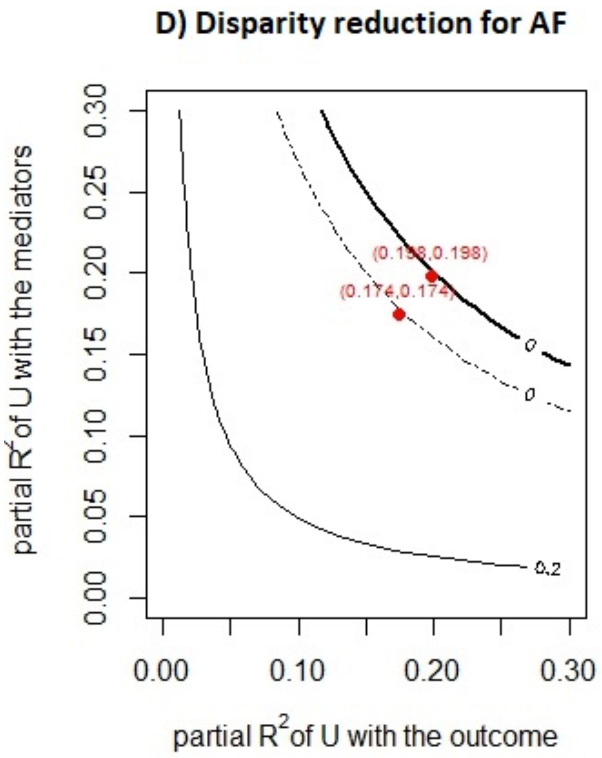
[2] BM: Black male, BF: Black female, HM: Hispanic male, and HF: Hispanic female.
The omitted variable is denoted as U. One sensitivity parameter is presented on the X-axis, indicating the association between U and math literacy. Another sensitivity parameter is presented on the Y-axis, indicating the association between U with the mediator (school SES or OTL). Both sensitivity parameters are represented as partial R^2 values. The solid line shows the combinations of the two sensitivity parameters that would result in zero disparity reduction. The dashed line shows the points where the 95% confidence intervals cover zero. The region below the dashed line is where the estimates are still significant despite omitted confounding. The region above the dashed line is when the significance of the estimate changes to non-significance due to omitted confounding. The dots in each figure represent the amount of confounding that will result in zero disparity reduction or change the significance of the estimate to non-significance, given the equal amount of confounding between two sensitivity parameters.


C) Disparity reduction for AM
partial R^2 of U with the mediators
partial R^2 of U with the outcome
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.00
0.10
0.20
0.30
(0.198,0.198)
(0.174,0.174)
0
0
0.2


D) Disparity reduction for AF
partial R^2 of U with the mediators
partial R^2 of U with the outcome
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.00
0.10
0.20
0.30
(0.198,0.198)
(0.174,0.174)
0
0
0.2